Bachelor of Arts (B.A.) Part–I Semester—I Examination

MATHEMATICS

Optional Paper–I

(M_1 : Algebra and Trigonometry)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All the questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Find the rank of matrix A by reducing it into the normal form, where

$$A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$ 6

(B) Show that the following system of linear equations is consistent and hence solve the system of linear equations :

$x + y + z = 3,\ x + 2y + 3z = 4,\ x + 4y + 9z = 6.$ 6

OR

(C) Find the eigen values of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

Also find the eigen vector corresponding to only one eigen value. 6

(D) Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ satisfies Cayley-Hamilton theorem and find A^{-1}. 6

UNIT–II

2. (A) Solve the equation $x^3 - 3x^2 + 4 = 0$, two of its roots being equal. 6

(B) If α, β, γ be the roots of the cubic $x^3 + px^2 + qx + r = 0$, then calculate the values of the symmetric functions :

(i) $\Sigma\alpha^2$

(ii) $\Sigma\alpha^2\beta^2$

(iii) $\Sigma \frac{1}{\alpha}$ 6

OR

(C) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method. 6

(D) Solve the biquadratic equation $x^4 + 12x - 5 = 0$ by Ferrari's method. 6

UNIT–III

3. (A) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$. Then prove that :

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$ and

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin (\alpha + \beta + \gamma)$. 6

(B) Find all the values of $(1)^{\frac{1}{4}}$. 6

OR

(C) Prove that :

(i) If $\sinh y = x$ then $y = \sinh^{-1} x = \log\left(x + \sqrt{x^2 + 1}\right)$ and

(ii) If $\cosh y = x$ then $y = \cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$ 6

(D) Prove that :

(i) $\text{Log}(-x) = (2n + 1)\pi i + \log x$ and

(ii) $\text{Log}(xi) = (2n + \frac{1}{2})\pi i + \log x$. 6

UNIT–IV

4. (A) Show that G = {1, –1, i, –i} is an abelian group of order 4 with respect to multiplication. 6

(B) Prove that intersection of two subgroups of a group is a subgroup. Give an example to show that union of two subgroups is not necessarily a subgroup of a group. 6

OR

(C) Prove that the order of a subgroup of finite group is a divisor of the order of the group. 6

(D) (i) Find whether a permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 3 & 1 & 7 \end{pmatrix} \text{ is even or odd.}$$

(ii) If $f = \begin{pmatrix} a_1 & a_2 & a_3 & \ldots & a_n \\ b_1 & b_2 & b_3 & \ldots & b_n \end{pmatrix}$, then prove that $f.f^{-1} = I$, where I is the identity permutation of degree n. 6

QUESTION–V

5. (A) Write the augmented matrix [A : B] for the system y + 2z = a, x + 2y + 3z = b, 3x + y + z = c. 1½

(B) Give that characteristic equation of the matrix :

$$A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix} \text{ is } A^2 - 2A - 3I = 0. \text{ Find } A^{-1}.$$ 1½

(C) Form an equation whose roots are 1, 2, 3. 1½

(D) Using Descarte's rule of signs show that an equation $x^3 - 9x^2 + 12x = 0$ has two positive real roots and one negative real root. 1½

(E) Prove that cos (ix) = cosh x. 1½

(F) Prove that log(–1) = πi. 1½

(G) Prove that the identity element of group is unique. 1½

(H) Find all the right cosets of subgroup H = {1, –1} in a multiplicative group G = {1, –1, i, –i}. 1½